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Notes accompanying Amicus Brief submitted to Wisconsin Supreme Court on Nov. 8, 2023

Note 1

A composite, or average, value for the number of voters supporting a party in a ward can be computed by averaging the number of votes cast for the party in the six elections that are considered. For example, if Party A received 400, 400, 500, 500, 500, and 500 votes in Ward *W* in the six elections, its composite vote total in Ward *W* is (400 + 400 + 500 + 500 + 500 + 500)/6 = 466.67. This value is then summed for all wards in a proposed district to give the predicted vote total for Party A in the district. This is then compared to Party B's predicted vote total in the district to predict who wins the district. Predictions at the district level are then aggregated to predict the total number of seats won by each party for the map at hand.

Note 2

Figures A-B below show the difference between a *past-the-post* and *fractional seats* approach to analyzing political fairness. Each figure considers Party A's share of the two-party vote in a single district. As shown in Figure A, *past-the-post* accounting allocates one seat to the party with more voters in a district no matter if the district is lopsided or closely contested. If the district is perfectly tied, each party is assumed to win 0.5 seats in it.

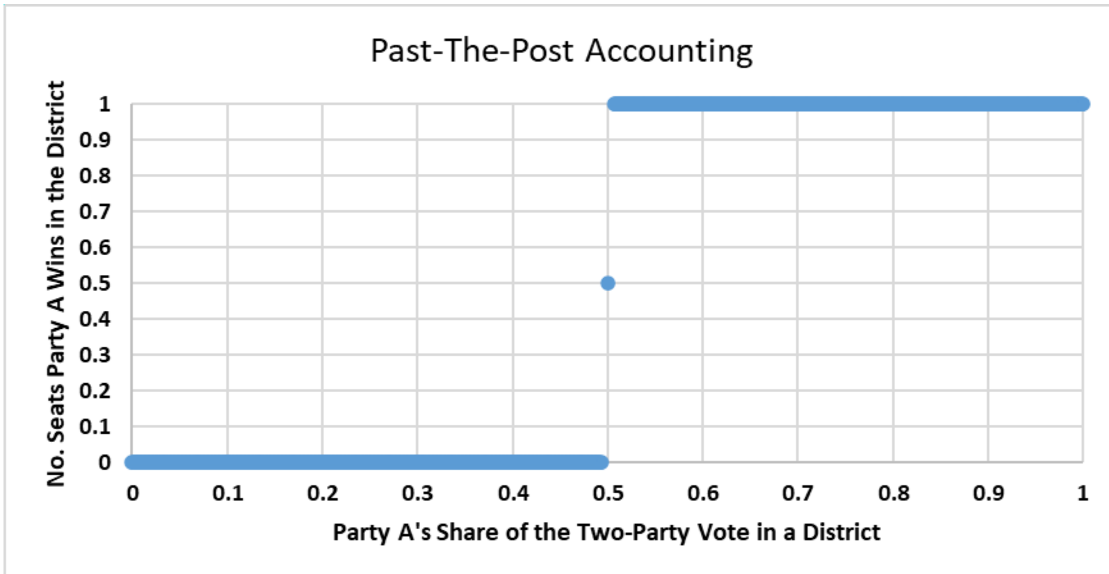

Figure A. In past-the-post accounting, a district is categorized as a complete loss (win) if a party has less (more) than 50% of the two-party vote in the district.

As shown in Figure B, *fractional seats* accounting assumes that a district is a total win or loss only if it is lopsided. If the district is competitive, each party is assumed to have a non-zero probability of winning it, i.e., a fractional predicted number of victories in it between 0 and 1. For example, DavesRedistricting.org assumes that a party with a two-party vote share of (50, 52, 54, 56, 58, 60) percent in a district has a (50.0, 69.1, 84.1, 93.3, 97.7, 99.4) percent chance of winning it.

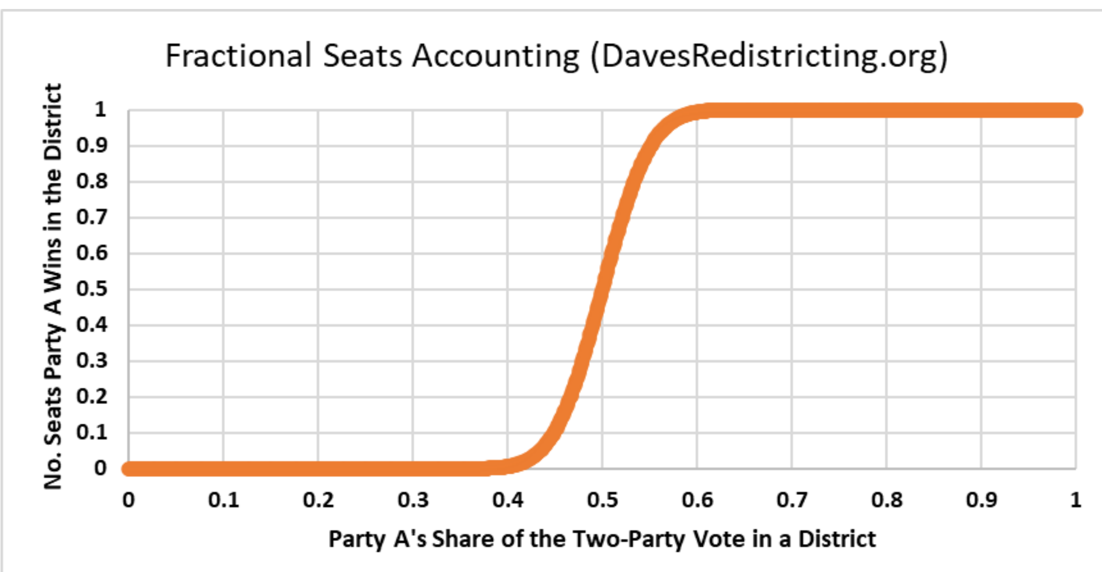

Figure B. In fractional seats accounting, a district is assumed to be a complete win or loss only if it is lopsided. If a district is competitive, each party is assumed to have a fractional, non-zero probability of winning it (i.e., a fractional, non-zero predicted number of seats it wins in the district).

Note 3

The following study considers the elections in all 54 state legislative chambers—39 houses and 15 senates—in which all seats in the chamber were up for election in November 2022 and there are no multi-member districts. In each of the 54 cases, the map used in the 2022 election was populated with 2016-2020 composite voting data from DavesRedistricting.org, and the predicted number of districts won by each party under the *fractional seats* and *past-the-post* approaches were compared to the actual number of districts won in the 2022 election. In 26 cases, the *fractional seats* prediction was better; in 14 cases the *past-the-post* prediction was better; and in 14 cases the two methods were equally good. Overall, the fractional seats approach was superior. The results for the 15 senate chambers are shown in Table A. The results for the 39 house chambers are shown in Table B.

Table A. Comparison of *fractional seats* and *past-the-post* accounting for elections in 15 state senate chambers in November 2022.

State Senate Analysis (DavesRedistricting.org)						
State	**Dem 2 party vote share (%)**	**Dem seats closest to proportional**	**Dem Fractional seats prediction**	**Dem Fractional seats prediction (rounded)**	**Dem Past-the-post prediction**	**Actual Dem seats won**,1**
Alabama	40.02	14	8.8	9*	9*	8
Arizona	48.87	15	13.96	14*	13	14
Connecticut	58.08	21	29.47	29*	31	24
Georgia	48.03	27	22.61	23*	23*	23
Idaho	33.87	12	4.79	5*	4	7
Maine	50.63	18	16.5	16	17*	22
Maryland	62.15	29	32.6	33*	33*	34
Massachusetts	61.39	25	35.49	35	37*	37
Michigan	51.87	20	19.82	20*	19	20
Minnesota	54.48	37	37.26	37*	38	34
New Hampshire	46.15	11	7.34	7	8*	10
New York	64.78	41	50.03	50*	51	42
North Carolina	49.43	25	22.31	22*	22*	20
Rhode Island	61.89	23	33.24	33*	34	33
South Dakota	37.11	13	3.39	*3	3*	4
			15 states	**7***	**3***	**5**
				cases where fractional seats prediction (rounded) is better	**cases where past-the-post prediction is better**	**cases where both predictions are equally good**

**https://en.wikipedia.org/wiki/2022_United_States_state_legislative_elections

Table B. Comparison of *fractional seats* and *past-the-post* accounting for elections in 39 state house chambers in November 2022.

State Assembly Analysis (DavesRedistricting.org)						
State	**Dem 2 party vote share (%)**	**Dem seats closest to proportional**	**Dem Fractional seats prediction**	**Dem Fractional seats prediction (rounded)**	**Dem Past-the-post prediction**	**Actual Dem seats won**,1**
Alabama	40.02	42	29.69	30	29*	28
Alaska	44.18	18	16.1	16	15*	11
Arkansas	35.21	35	17.19	17*	16	18
California	64.24	51	65.82	66*	67	62
Colorado	54.48	35	40.65	41	44*	46
Connecticut	58.08	88	114.57	115*	116	98
Delaware	59.95	25	29.69	30*	30*	26
Florida	48.37	58	50.52	51	48*	35
Georgia	48.03	86	77.93	78	79*	79
Hawaii	68.27	35	50.14	50*	51	45
Idaho			2x senate	10*	8	11
Illinois	58.17	69	81.63	82*	83	78
Indiana	43.47	43	29.55	30*	29	30
Iowa	45.15	45	33.81	34*	32	36
Kansas	41.86	52	37.96	38*	37	40
Kentucky	40.45	40	20.29	20*	19	20
Maine	50.63	76	69.38	69*	69*	82
Massachusetts	61.39	98	131.64	132	133*	134
Michigan	51.87	57	55.12	55	56*	56
Minnesota	54.48	73	74.98	75*	77	70
Missouri	42.78	70	51	51*	50	52
Montana	43.64	44	40.25	40*	50	32
Nevada	51.45	22	27.77	28*	28*	28
New Mexico	56.09	39	47.1	47*	47*	45
New York	64.78	97	114.96	115	113*	102**
North Carolina	49.43	59	55.34	55*	57	49
Ohio	46.38	46	36.24	36*	42	31
Oklahoma	33.63	34	15.19	15*	14	20
Oregon	57.5	34	38.69	39*	39*	35
Pennsylvania	52.51	107	105.9	106*	106*	102
Rhode Island	61.89	46	64.92	65*	64	65
South Carolina	43.16	54	38.58	39*	39*	36
Tennesse	38.71	38	23.83	24*	23	24
Texas	46.22	69	65.72	66*	66*	64
Utah	32.97	25	11.04	11	12*	14
Washington			2x senate	63	62*	58
West Virginia	33.48	33	8.06	8*	6	12
Wisconsin	50.68	50	39.7	40	39*	35
Wyoming	27.54	17	5.49	5*	5*	5
			39 states	**19***	**11***	**9**
				cases where fractional seats prediction (rounded) is better	**cases where past-the-post prediction is better**	**cases where both predictions are equally good**

Note 4

A study was conducted which simulated 1,000,000 elections for each of 200 Wisconsin Assembly maps created by the *FastMap* algorithm. The simulations used the assumptions at DavesRedistricting.org, namely that a party with a two-party vote share of (50, 52, 54, 56, 58, 60) percent in a district has a (50.0, 69.1, 84.1, 93.3, 97.7, 99.4) percent chance of winning it (Figure B in Note 2).

Results are shown in Figure C. The figure contains 400 dots, one for each party's outcome in each map. The figure shows that a party's chances of winning a majority of seats in the Wisconsin Assembly (Y axis) is highly sensitive to the (fractional) predicted number of seats it wins for the map (X axis). According to the figure, increasing a party's (fractional) predicted seat total in the Wisconsin Assembly from 49 to 50 increases its chances of controlling the chamber from about 40% to 60%. And increasing its predicted seat total from 48 to 51 increases its chances of controlling the chamber from about 25% to 75%. Further, a party that is predicted to win 45 or fewer fractional seats has basically no chance of controlling the chamber.

These results highlight the importance of zeroing in on *strict proportionality* in a closely contested state such as Wisconsin. Hence, *the Court should strongly prioritize proportionality (Criterion 5) in the weighting scheme*.

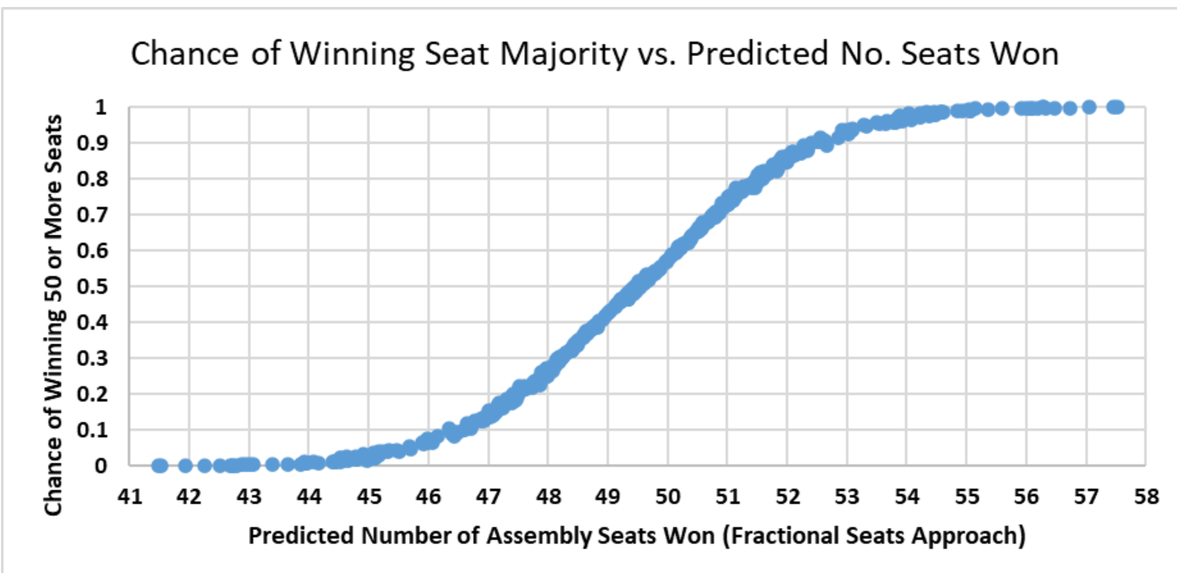

Figure C. Plot of a party's chances of winning a majority of Wisconsin Assembly seats versus the party's predicted number of assembly seats won using the fractional seats approach.

Note 5

Wisconsin's population was 5,893,718 according to the 2020 U.S. Census. Because Wisconsin has 33 senate districts and 99 assembly districts, the ideal population for each senate district is 178,598 and the ideal population for each assembly district is 59,533. Table C presents the population deviation scores for districts in map *155#176*. Courts outside Wisconsin generally presume that a state legislative plan is constitutional if it has an overall range in deviation of 10% or less. Wisconsin, however, has a tradition of adopting maps with an overall range in deviation of 2% or less. According to Table C, map *155#176* has a 1.98% (1.52%) range in population deviation in the assembly (senate), so it meets this requirement.

Table C. Analysis of population deviation in map *155#176*

	Deviation from Ideal Population	**Persons**	**Percent**
Assembly	Mean Deviation	291	0.489
	Largest Positive Deviation	584	0.981
	Largest Negative Deviation	–592	–0.994
	Overall Range in Deviation	± 1176	± 1.975 (i.e., 1.98%)
	Deviation from Ideal Population	**Persons**	**Percent**
Senate	Mean Deviation	587	0.329
	Largest Positive Deviation	1405	0.787
	Largest Negative Deviation	–1305	–0.731
	Overall Range in Deviation	± 2710	± 1.517 (i.e., 1.52%)